Guaranteed-cost H_∞ Observer Gain for Under-Tendon-Driven Prosthetic Fingers

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I. Introduction

- Estimation of angular displacement and velocity for under-tendon-driven prosthetic fingers based on current measurements.
- Discrete-Time H_∞ Full-State Observer Characterization.
- Robust observer gain matrix obtained using Linear Matrix Inequalities methods.
- Convex optimization problem solved using semidefinite programming.
- Implemented on the embedded controller of the Galileo Hand, an anthropomorphic and affordable upper-limp prosthesis [1].

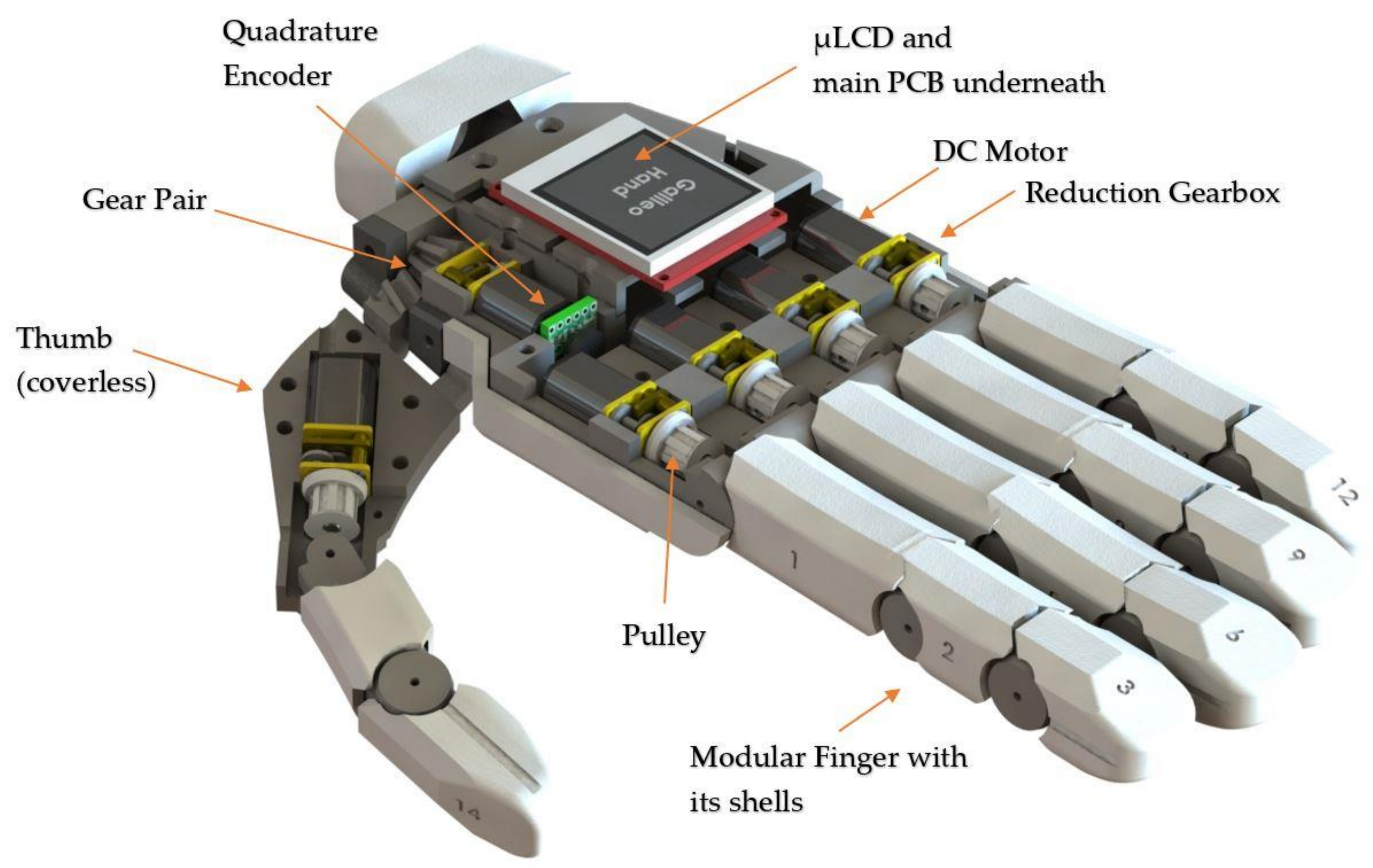
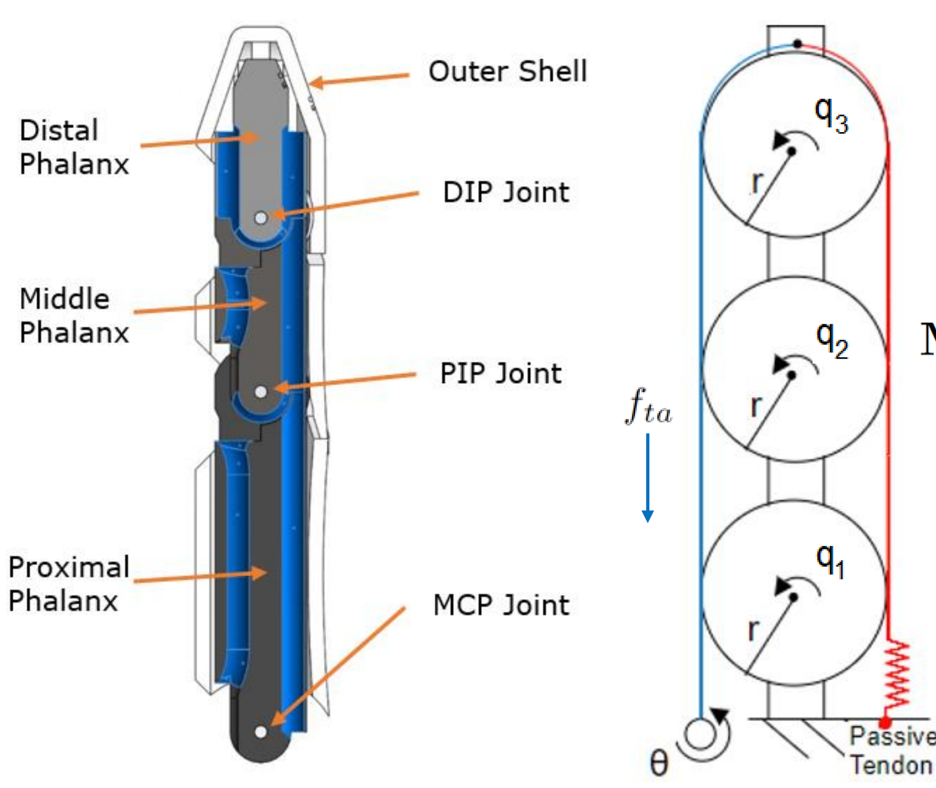


Fig.1: Mechanical design of the Galileo Hand.

II. The Under-tendon-driven Machine

The under-tendon-driven machine [2] modeled as a linear mass-spring system:



$$\mathbf{q} = \mathbf{J}_j^{+}\left[\, \boldsymbol{l} - \boldsymbol{l}_0 - \mathbf{J}_a\boldsymbol{\theta} \,\right] + \mathbf{q}_0$$

$$\mathbf{M}\ddot{\mathbf{q}} + \left[\frac{1}{2}\dot{\mathbf{M}} + \mathbf{S} + \mathbf{B}_0\right]\dot{\mathbf{q}} + \mathbf{G}_g\mathbf{q} = \boldsymbol{\tau}$$

$$J_m\ddot{\theta} + b\dot{\theta} + r_p f_{ta} = \tau_m$$

$$\tau_m = \eta G_r k_t i_a$$

Fig.2: Mechanical design for the fingers.

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 \\ -\frac{k_e r_p^2}{J_m} & -\frac{b}{J_m} & \frac{\eta G_r k_t}{J_m} \\ 0 & -\frac{k_t}{L_a} & -\frac{R_a}{L_a} \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{L_a} \end{bmatrix} u$$

$$\mathbf{y} = [0\ 0\ 1]\,\mathbf{x}$$

$$\mathbf{x} = \begin{bmatrix} \theta & \dot{\theta} & i_a \end{bmatrix}^T$$

III. Discrete-time H_∞ Full-state Observer

An observer-based filter was designed considering noise components as follows

$$\mathbf{x}_{k+1} = \mathbf{A}\mathbf{x}_k + \mathbf{B}_1\mathbf{u}_k + \mathbf{B}_2\mathbf{w}_k$$

$$\mathbf{y}_k = \mathbf{C}\mathbf{x}_k + \mathbf{D}_1\mathbf{v}_k + \mathbf{D}_2\mathbf{w}_k$$

$$\hat{\mathbf{x}}_{k+1} = \mathbf{A}\hat{\mathbf{x}}_k + \mathbf{B}_1\mathbf{u}_k - \mathbf{K}(\mathbf{y}_k - \hat{\mathbf{y}}_k)$$

The guaranteed-cost observer gain can be obtained by solving the following optimization problem

$$\min_{\mathbf{Z},\mathbf{G},\mathbf{P}=\mathbf{P}^T>0} \gamma$$

subjected to the following LMI

$$\begin{bmatrix} \mathbf{P} & \mathbf{A}^T\mathbf{G} + \mathbf{C}^T\mathbf{Z}^T & \mathbf{0}_{n\times s} & \mathbf{0}_{n\times s} & \mathbf{C}^T \\ \star & \mathbf{G} + \mathbf{G}^T - \mathbf{P} & \mathbf{G}^T\mathbf{B}_2 + \mathbf{Z}\mathbf{D}_2 & \mathbf{Z}\mathbf{D}_1 & \mathbf{0}_{n\times q} \\ \star & \star & \mathbf{I}_s & \mathbf{0}_{s\times s} & \mathbf{D}_2^T \\ \star & \star & \star & \mathbf{I}_s & \mathbf{D}_1^T \\ \star & \star & \star & \star & \gamma^2\mathbf{I}_q \end{bmatrix} > 0$$

$$\mathbf{K} = (\mathbf{G}^T)^{-1}\mathbf{Z}$$

IV. Results

The experiments to test and validate the methods were carried out using the index finger of the Galileo Hand.

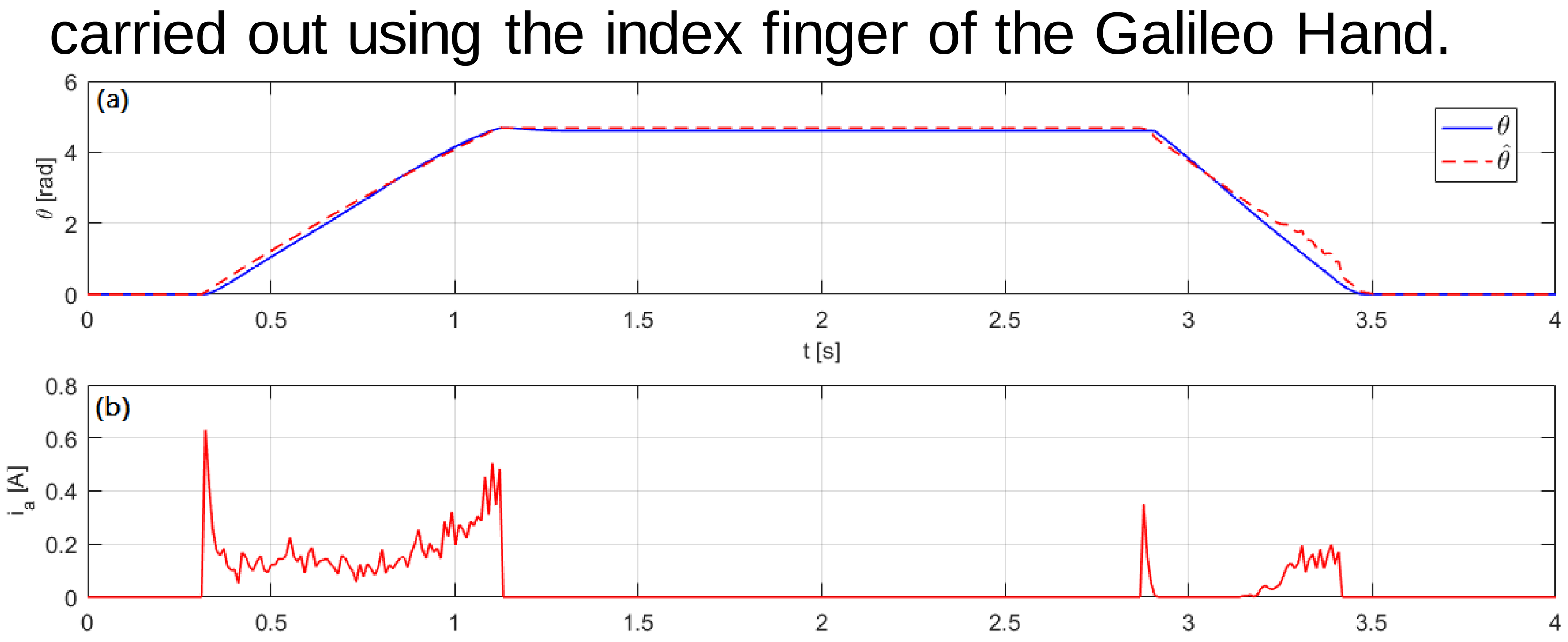


Fig.3: Extension and flexion processes of the finger.

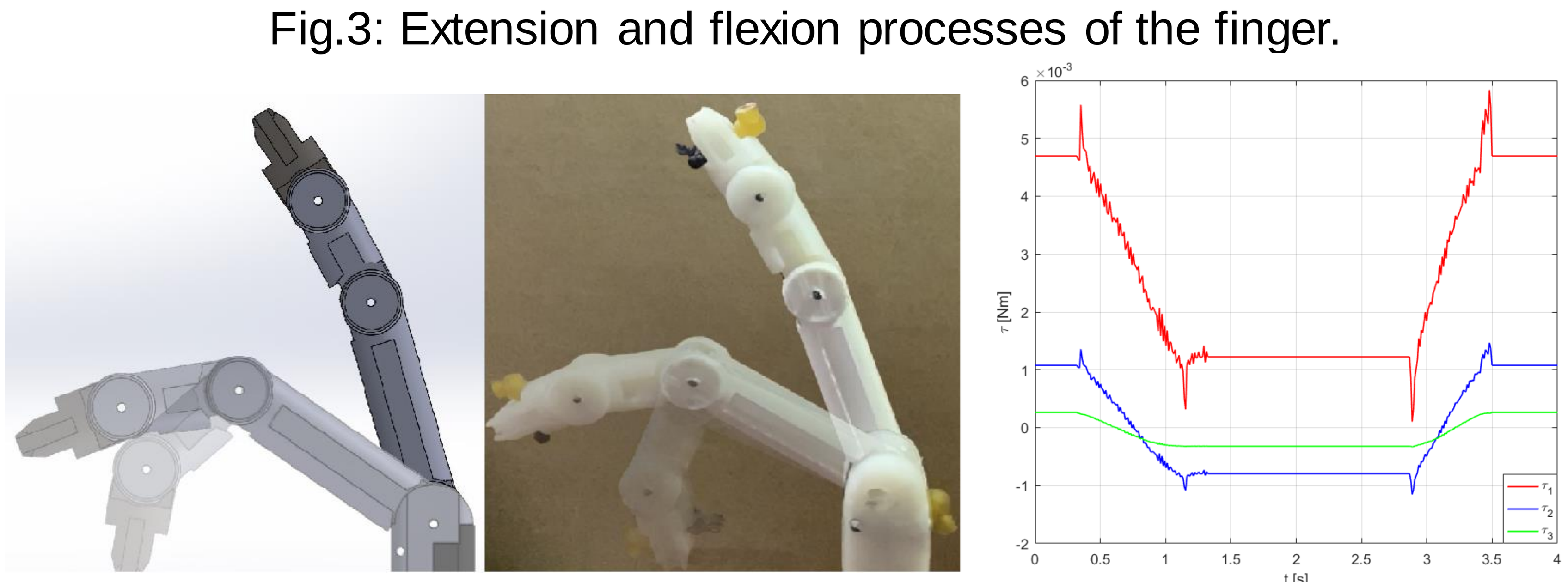


Fig.4: Estimation of generalized coordinates and joint torques

V. Conclusions

- Successful alternative for full state estimation for an under-tendon-driven prostheses' fingers.
- Sensorless operation of the fingers.
- $RMSE$ for θ is about 0.1394 rad.
- Sufficient for the apt fulfillment of ADLs..
- This leads to a reasonable estimation of the dynamics of each finger.
- Polytopic representation for LTV and non-linear systems.
- Takagi-Sugeno modeling for fuzzy systems.
- Sum of Squares (SOS) approach using Polynomial Programming.

VI. References

[1] J. Fajardo, V. Ferman, D. Cardona, G. Maldonado, A. Lemus, and E. Rohmer, "Galileo hand: An anthropomorphic and affordable upper-limb prosthesis," IEEE Access, vol. 8, pp. 81 365–81 377, 2020.
[2] R. Ozawa, K. Hashirii, and H. Kobayashi, "Design and control of underactuated tendon-driven mechanisms," in 2009 IEEE International Conference on Robotics and Automation. IEEE, 2009, pp. 1522–1527.